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I. Relaxation model

Damping of flexible structures by active control depends strongly on the type and design of the applied control methodology, where in particular collocated strategies have been widely implemented due to robustness and simplicity [1].

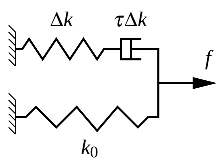

Fig. 1. Relaxation model.

The damping introduced by the structural control should target the dominant vibration mode(s) in the low-frequency range, without exciting higher modes. This suggests a control equation in the form of a low-pass filter. In mechanics of materials the so-called relaxation models are used to describe the relation between stress (force) and strain (motion) amplitudes over a wide frequency range [2, 3]. Figure 1 shows the simple relaxation model with two springs placed in parallel through a viscous dashpot. For very slow motion only the bottom spring is activated, determining k_0 as the low-frequency or fully relaxed stiffness. For very fast motion the dashpot acts as a rigid link, whereby the high-frequency stiffness becomes the sum of the two springs: $k_\infty = k_0 + \Delta k$. Thus, the full model can be written as

$$i\omega\tau(f - k_\infty x) + (f - k_0 x) = 0 \tag{1}$$

where the time scale $\omega\tau$ determines the transition between the two limiting regimes. The frequency relation (1) can be written as $f = H(\omega)x$, and Fig. 2 shows the real and imaginary part of the transfer function. Energy dissipation requires a positive imaginary part and causality then implies $\Delta k > 0$.

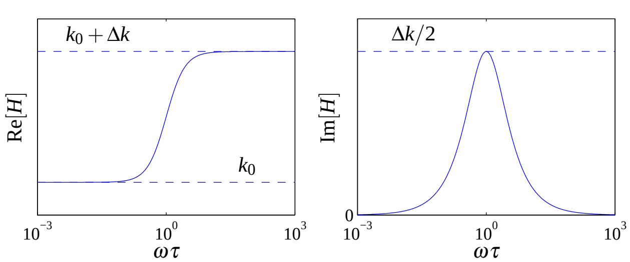

Fig. 2. Real imaginary part of frequency transfer function.

In mechanical (passive) models the low-frequency stiffness k_0 is non-negative. However, the aim of the present paper is to illustrate the relaxation model for control of flexible structures, whereby active forces may realize negative device stiffness, i.e. $k_0 < 0$, as long as the stability of the system is secured.

II. Structural Control

Consider a control force f acting on a flexible structure. The equation of motion can be written in the standard form

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = -\mathbf{w}\, f + \mathbf{F} \tag{2}$$

where the connectivity vector $\mathbf{w}$ represents the location of the control force on the structure. The control force is determined by the relaxation model in (1). When the change in stiffness Δk is introduced as gain factor the control equation can be written as

$$\tau\dot{f} + f = (k_0 + \Delta k)\tau\dot{x} + k_0\, x \tag{3}$$

The low-frequency stiffness k_0 can be negative, and the corresponding stability limit can be determined as

$$k_0^{stab} = -\left(\mathbf{w}^T\mathbf{K}^{-1}\mathbf{w}\right)^{-1} \tag{4}$$

which simply states that any negative stiffness of the control should not exceed the inverse of the structural flexibility at the actuator location. Figure 3 shows a root locus plot for mode 1 of a 10-storey shear frame structure where the control force acts on the first floor. As seen in Fig. 3 improved damping performance is obtained for decreasing values of the time scale $\omega\tau$ and negative values of k_0.

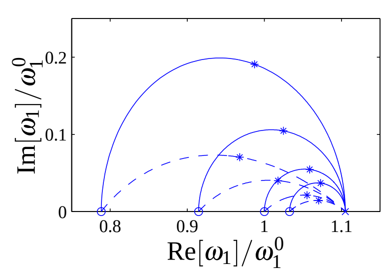

Fig. 3. Root locus plot for mode 1: $\omega_1^0\tau = 0.01$ (solid), 1.00 (dashed) and $k_0/k_0^{stab} = -0.75, -0.50, 0.00, 0.50$.

References

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